SR01 RELATIVITY

SPH4U

CH 11 (KEY IDEAS)

- State Einstein’s two postulates for the special theory of relativity.
- Describe Einstein’s thought experiments demonstrating relativity of simultaneity, time dilation, and length contraction.
- State the laws of conservation of mass and energy, using Einstein’s mass–energy equivalence.
- Conduct thought experiments involving objects travelling at different speeds, including those approaching the speed of light.

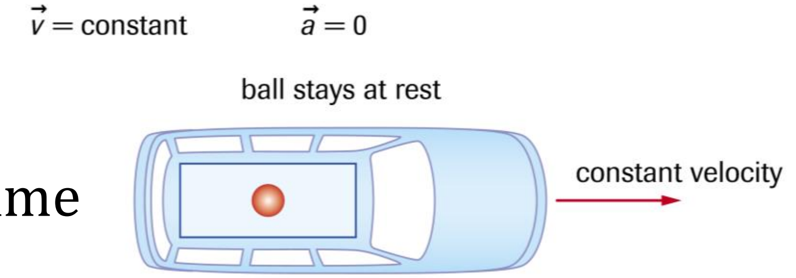
EQUATIONS

- Time Dilation for Moving Objects

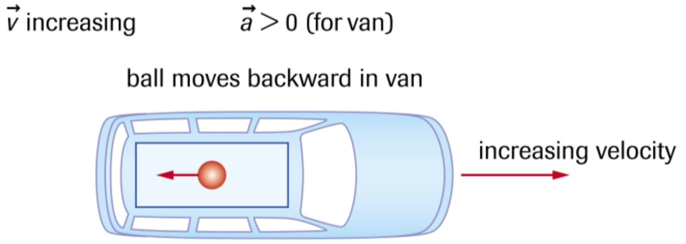
$$\Delta t_m = \frac{\Delta t_s}{\sqrt{1 - \frac{v^2}{c^2}}}$$

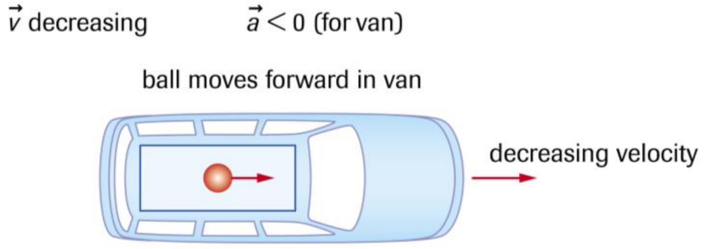
FRAMES OF REFERENCE

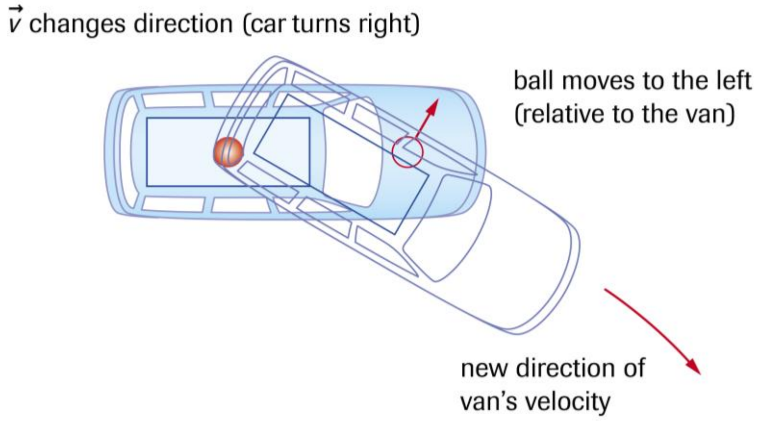
- **Inertial Frame of Reference:** a frame of reference in which Newton's <u>law of inertia</u> holds
  - Newton's laws only hold in this type of reference frame
  - $\vec{F} = 0$ for the frame of reference, not the object



- **Noninertial Frame of Reference:** frame of reference that is accelerating relative to an inertial frame
  - Ex: an accelerating vehicle
  - RECALL: a vehicle can have a constant speed and still accelerate (circular motion)





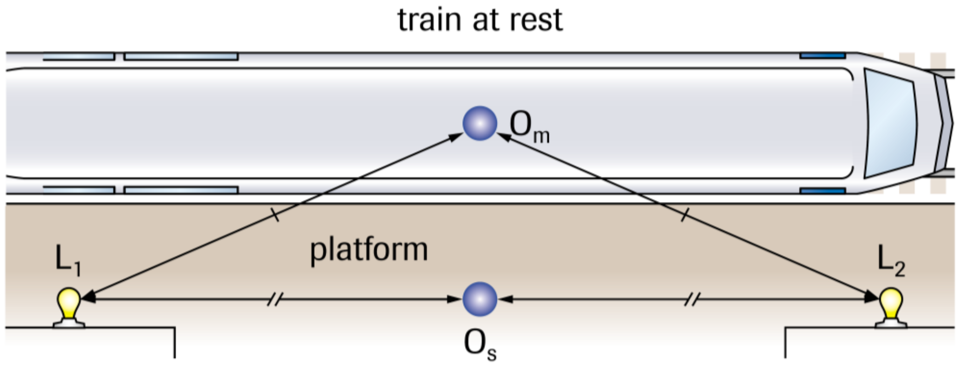


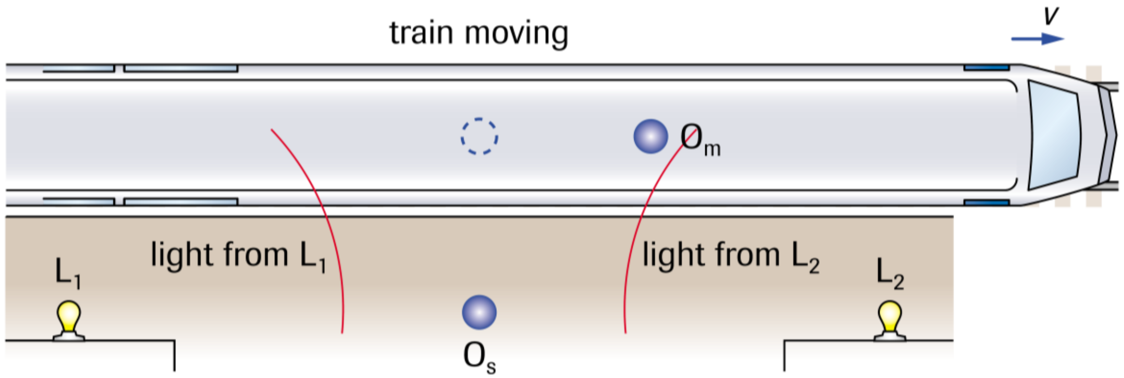
SPECIAL THEORY OF RELATIVITY

- The speed at which an object is travelling depends on the frame of reference of the observer
- Does the speed of light depend on the frame of reference of the observer? No, light is special since it doesn't need to travel through a medium.
- **Special Theory of Relativity**
    1. <u>The relativity principle:</u> all the laws of physics are valid in all inertial frames of reference.
    2. <u>The constancy of the speed of light:</u> light travels through empty space with a speed of $c = 3.00 \times 10^8$ m/s, relative to all inertial frames of reference.

SIMULTANEITY

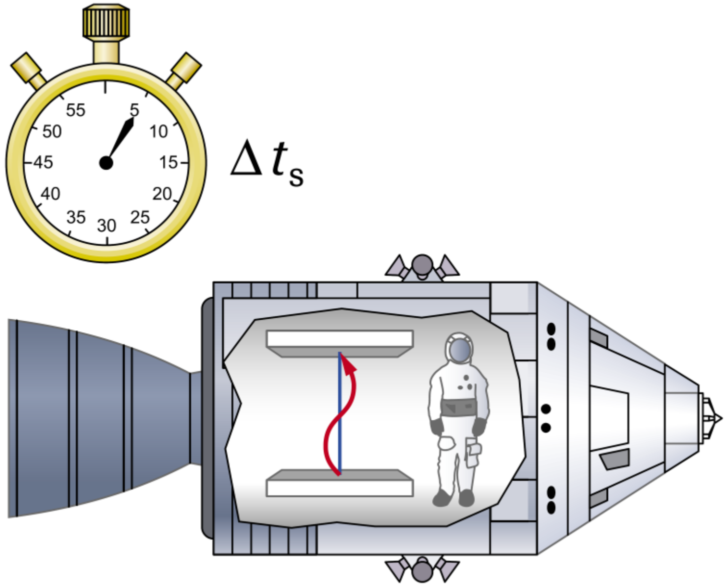
- **Simultaneity:** the occurrence of two or more events at the same time
- Two events that are simultaneous in one frame of reference are in general not simultaneous in a second frame moving with respect to the first
  - simultaneity is not an absolute concept





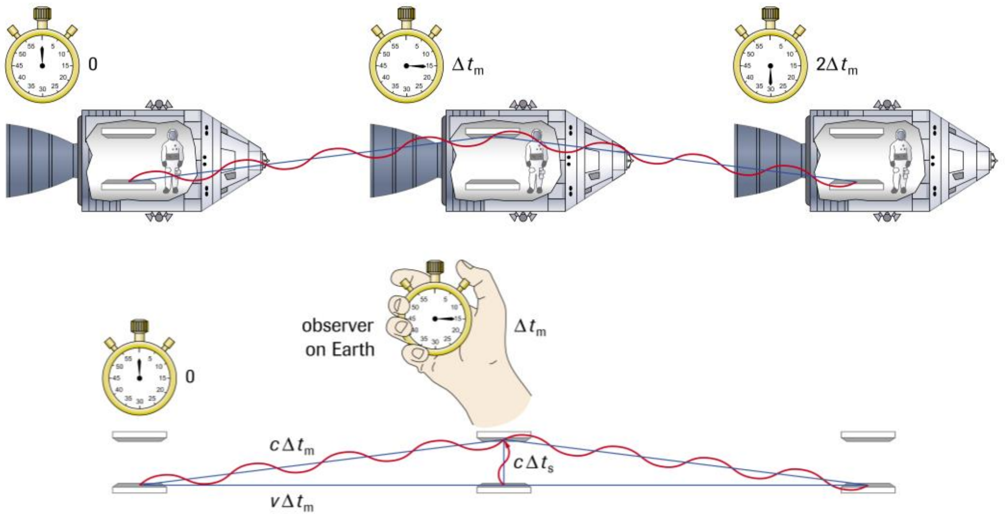
TIME DILATION

- Time is not absolute
- Consider a set of parallel mirrors on a spacecraft with a beam of light passing between them
- The astronaut on the spacecraft will measure the time it takes to pass between the mirrors whether the spacecraft is moving or stationary



TIME DILATION – CONT.

- An observer on Earth will measure the time differently depending on the speed of the spacecraft relative to Earth
- The light will take longer to move between the mirrors from their frame of reference as it has a longer distance to travel
    - Recall: the speed of light is absolute



TIME DILATION – CONT.

- Using the Pythagorean Theorem, the equation for time dilation for moving objects is

$$\Delta t_m = \frac{\Delta t_s}{\sqrt{1 - \frac{v^2}{c^2}}}$$

  - Δt_m – time interval for an observer moving at speed v relative to the sequence of events [s]
  - Δt_s – time interval for an observer stationary relative to the sequence of events [s]
  - v – speed of the observer [m/s]
  - c – the speed of light [m/s]

TIME DILATION – CONT.

- **Proper Time ($\Delta \boldsymbol{t_s}$) [s]:** the time interval between two events measured by an observer who sees the events occur at one position
- **Time Dilation:** the slowing down of time in a system, as seen by an observer in motion relative to the system
- $\Delta t_s < \Delta t_m$ by a factor of $\frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$, $0 < v < c$
  - Δt_s is only a real number if $1 - \frac{v^2}{c^2} > 0$, or $v < c$
  - An object cannot have a speed equal to or greater than the speed of light

PROBLEM 1

An astronaut whose pulse frequency remains constant at 72 beats/min is sent on a voyage. What would her pulse beat be, relative to Earth, when the ship is moving relative to Earth at (a) $0.10c$ and (b) $0.90c$?

PROBLEM 1 – SOLUTIONS

$c = 3.0 \times 10^8$ m/s

pulse frequency $=$ 72 beats/min

pulse period at relativistic speed $= \Delta t_m = ?$

The pulse period measured on the spaceship (where pulses occur in the same position) is

$$\Delta t_s = \frac{1}{72\ \text{min}^{-1}}$$

$$\Delta t_s = 0.014\ \text{min}$$

(a) At $v = 0.10c$,

$$\Delta t_m = \frac{\Delta t_s}{\sqrt{1 - \dfrac{v^2}{c^2}}}$$

$$= \frac{0.014\ \text{min}}{\sqrt{1 - \dfrac{(0.10c)^2}{c^2}}}$$

$$= \frac{0.014\ \text{min}}{0.995}$$

$$\Delta t_m = 0.014\ \text{min, to 2 significant digists}$$

$$f = \frac{1}{T}$$

$$= \frac{1}{0.014\ \text{min}}$$

$$f = 72\ \text{beats/min}$$

The pulse frequency at $v = 0.10c$, relative to Earth, is to two significant digits unchanged, at 72 beats/min.

PROBLEM 1 – SOLUTIONS CONT.

(b) At $v = 0.90c$,

$$\Delta t_m = \frac{0.014 \text{ min}}{\sqrt{1 - \frac{(0.90c)^2}{c^2}}}$$

$$= \frac{0.014 \text{ min}}{0.436}$$

$$\Delta t_m = 0.032 \text{ min}$$

$$f = \frac{1}{T}$$

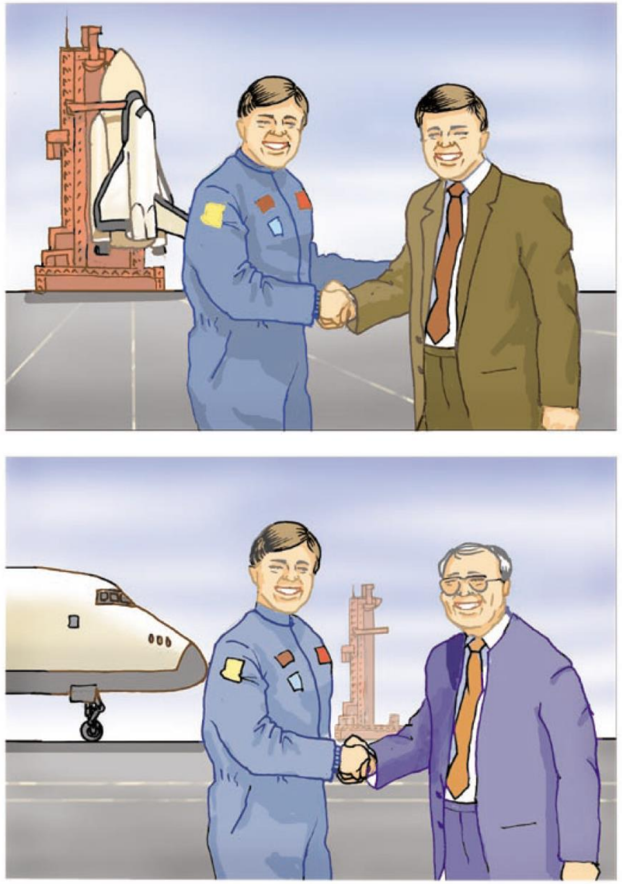
$$= \frac{1}{0.032 \text{ min}}$$

$$f = 31 \text{ beats/min}$$

The pulse frequency at $v = 0.90c$, relative to Earth, is 31 beats/min. This result is much lower than the 72 beats/min that the astronaut would measure for herself on the ship.

THE TWIN PARADOX

- Consider a twin travelling to a star and back at a speed approaching c while their twin stays on Earth
- From the change in heartbeat found in Problem 1, we can deduce that the travelling twin will age less than the twin left on Earth
- The reverse is not true in the travelling twin's perspective because they are not in an inertial reference frame
  - Acceleration occurs in order to turn around, so the situation is not symmetrical



LENGTH CONTRACTION

- Similar to time, length is not absolute
- **Proper Length (L_s) [m]:** the length, in an inertial frame, of an object stationary in that frame
- **Length Contraction:** the shortening of distances in a system, as seen by an observer in motion relative to that system

LENGTH CONTRACTION – CONT.

- From $v = \frac{d}{\Delta t}$, $L_s = v\Delta t_m$ and $L_m = v\Delta t_s$, which leads to

$$L_m = L_s\sqrt{1 - \frac{v^2}{c^2}}$$

  - L_m – distance measured by an observer in an inertial frame of reference [m]
  - L_s – proper distance [m]
  - v – speed of the object [m/s]
  - c – speed of light [m/s]

PROBLEM 2

A UFO heads directly for the centre of Earth at $0.500c$ and is first spotted when it passes a communications satellite orbiting at 3.28×10^3 km above the surface of Earth. What is the altitude of the UFO at that instant as determined by its pilot?

PROBLEM 2 – SOLUTIONS

Before solving a problem in special relativity, we must work out which lengths or durations are proper. In this case, the UFO pilot is determining the length separating two events in a process *moving* through his frame (the first event being the arrival at the UFO of the satellite, the second the arrival at the UFO of the surface of Earth). The length that the pilot is determining is thus *not* a proper length so it can appropriately be called L_m. On the other hand, the given length of 3.28×10^3 km is the length separating two events in a process *stationary* in the frame to which it is referred (the first event being the arrival of the UFO at the satellite, the second the arrival of the UFO at the surface of Earth). The given length is thus a proper length so it can appropriately be called L_s.

PROBLEM 2 – SOLUTIONS

$v = 0.500c$

$L_s = 3.28 \times 10^3$ km

$L_m = ?$

$$L_m = L_s\sqrt{1 - \frac{v^2}{c^2}}$$

$$= (3.28 \times 10^3 \text{ km})\sqrt{1 - \frac{(0.500c)^2}{c^2}}$$

$$L_m = 2.84 \times 10^3 \text{ km}$$

The altitude as observed from the UFO is 2.84×10^3 km.
Since the speed of the UFO is fairly slow, as far as relativistic events are concerned, the distance contraction is relatively small.

PROBLEM 3

A spaceship travelling past Earth with a speed of $0.87c$, relative to Earth, is measured to be 48.0 m long by observers on Earth. What is the proper length of the spaceship?

PROBLEM 3 – SOLUTIONS

Since the spaceship is moving relative to the observers on Earth, 48.0 m represents L_m.

$v = 0.87c$

$L_m = 48.0\text{ m}$

$L_s = ?$

$$L_m = L_s\sqrt{1 - \frac{v^2}{c^2}}$$

$$L_s = \frac{L_m}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$= \frac{48.0\text{ m}}{\sqrt{1 - \frac{(0.87c)^2}{c^2}}}$$

$$L_s = 97.35\text{ m, or } 97.4\text{ m}$$

The proper length of the spaceship is 97.4 m.

RELATIVISTIC MOMENTUM

- Momentum is not absolute
- **Rest Mass:** mass measured at rest, relative to the observer
- Relativistic Momentum

$$p = \frac{mv}{\sqrt{1 - \frac{v^2}{c^2}}}$$

  - p – magnitude of the relativistic momentum [kg m/s]
  - m – rest mass of the object [kg]
  - v – speed of the object relative to an observer at rest [m/s]
  - c – speed of light [m/s]

PROBLEM 4

Linear accelerators accelerate charged particles to nearly the speed of light (**Figure 5**). A proton is accelerated to 0.999 994c.

(a) Determine the magnitude of the relativistic momentum.

(b) Make an order-of-magnitude comparison between the relativistic and the nonrelativistic momenta.

PROBLEM 4 – SOLUTIONS

(a) $v = 0.999\ 994c$

$m = 1.67 \times 10^{-27}$ kg (from Appendix C)

$p = ?$

$$p = \frac{mv}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$= \frac{(1.67 \times 10^{-27}\ \text{kg})(0.999\ 994c)}{\sqrt{1 - \frac{(0.999\ 994c)^2}{c^2}}}$$

$$= \frac{5.010 \times 10^{-18}\ \text{kg·m/s}}{3.4641 \times 10^{-3}}$$

$$p = 1.45 \times 10^{-15}\ \text{kg·m/s}$$

The magnitude of the relativistic momentum of the proton is 1.45×10^{-15} kg·m/s. (This value agrees with the value that can be measured in the accelerator.)

PROBLEM 4 – SOLUTIONS

(b) The magnitude of the nonrelativistic momentum, given by the classical Newtonian equation, is

$$
\begin{aligned}
p &= mv \\
&= (1.67 \times 10^{-27}\ \text{kg})(0.999\,994c) \\
&= (1.67 \times 10^{-27}\ \text{kg})(0.999\,994)(3.00 \times 10^{8}\ \text{m/s}) \\
p &= 5.01 \times 10^{-19}\ \text{kg}\cdot\text{m/s}
\end{aligned}
$$

The nonrelativistic momentum is 5.01×10^{-19} kg·m/s. The relativistic momentum is more than three orders of magnitude larger than the nonrelativistic momentum.

SUMMARY – FRAMES OF REFERENCE AND RELATIVITY

- Any frame of reference in which the law of inertia holds is called an inertial frame of reference.
- A noninertial frame is one that is accelerating relative to an inertial frame.
- The laws of Newtonian mechanics are only valid in an inertial frame of reference and are the same in all inertial frames of reference.
- In Newtonian mechanics, no experiment can identify which inertial frame is truly at rest and which is moving. There is no absolute inertial frame of reference and no absolute velocity.
- Michelson and Morley's interferometer experiment showed that the ether does not exist.
- The two postulates of the special theory of relativity are:
    1. all laws of physics are the same in all inertial frames of reference;
    2. light travels through empty space with a speed of $c = 3.00 \times 10^8$ m/s in all inertial frames of reference.
- Simultaneity of events is a relative concept.

SUMMARY – RELATIVITY OF TIME, LENGTH, AND MOMENTUM

- Proper time Δt_s is the time interval separating two events as seen by an observer for whom the events occur at the same position.
- Time dilation is the slowing down of time in a system, as seen by an observer in motion relative to the system.
- The expression $\Delta t_m = \frac{\Delta t_s}{\sqrt{1-\frac{v^2}{c^2}}}$ represents time dilation for all moving objects.
- Time is not absolute: both simultaneous and time duration events that are simultaneous to one observer may not be simultaneous to another; the time interval between two events as measured by one observer may differ from that measured by another.
- Proper length L_s is the length of an object, as measured by an observer at rest relative to the object.
- Length contraction occurs only in the direction of motion and is expressed as $L_m = L_s\sqrt{1-\frac{v^2}{c^2}}$
- The magnitude p of the relativistic momentum increases as the speed increases according to the relationship $p = \frac{mv}{\sqrt{1-\frac{v^2}{c^2}}}$.
- The rest mass m of an object is its mass in the inertial frame in which the object is at rest and is the only mass that can be uniquely defined.
- It is impossible for an object of nonzero rest mass to be accelerated to the speed of light.

PRACTICE

Readings

- Section 11.1 (pg 562)
- Section 11.2 (pg 569)

Questions

- pg 568 #1-3
- pg 579 #1,3,5,7,10